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Low-Energy Consequences of High-Energy Quantum Chaos

F. Winterberg[1]

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It is shown that special and general relativity can be derived as low-energy approximations from the nonrelativistic quantum chaos of a vortex sponge. If the average distance of separation between the filaments of the vortex sponge is chosen to be $\sim 10^{-30}$ cm, a value suggested by the grand unified scale of elementary particle theories, the vortex core radius becomes about equal to the Planck length. The model permits a simple explanation of the phenomenon of charge, and in conjunction with a hypothesis by A. D. Sakharov, can explain Dirac spinors.

1. INTRODUCTION

Quantum field theory predicts a divergent ω^3-frequency spectrum of the vacuum zero-point energy fluctuations, the only spectrum which is invariant under a Lorentz transformation. Since a divergent vacuum energy is obviously a physical impossibility (it would lead to large gravitational fields), there must be a cutoff, most likely at some very high energy. The existence of such a cutoff would still make the vacuum energy very large, but this large energy might be compensated by a large cosmological constant. In any case, Lorentz invariance would be destroyed above the contemplated cutoff energy. Gauge theories of elementary particles suggest a unification of all interactions at a very high energy, estimated to be around 10^{16} GeV. It is therefore a plausible hypothesis that this energy coincides with the conjectured cutoff energy, and that at and above this energy the fundamental kinematic symmetry in nature might be the Galilei group, broken below this energy into the Lorentz group.

We therefore make here the assumption that the vacuum, at and above $\sim 10^{16}$ GeV, can be described by a superfluid state having a large number

[1]Desert Research Institute, University of Nevada System, Reno, Nevada 89506.

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of quantized vortex filaments. Such a configuration is sometimes called a vortex sponge. It was studied by Thomson (1887) as a mechanical aether model to describe the properties of electromagnetic waves derived from Maxwell's equations. To establish a connection to the conjectured unification energy of all interactions at $\sim 10^{16}$ GeV, we assume that the average distance between the vortex filaments is about $r_1 \sim 10^{-30}$ cm, with $mc^2 \sim \hbar c / r_1 \sim 10^{16}$ GeV.

2. FORMATION OF THE VORTEX SPONGE

It is known from many experiments in classical fluid dynamics that for a value of the Reynolds number $\mathrm{Re} \sim 10^6$ ($\mathrm{Re} = rv/\nu$, ν = kinematic viscosity) the frictional drag is minimized. Frictional dissipation of a fluid leads to solutions for the velocity field of the form

$$v \propto \exp(-\mathrm{const} \cdot \nu t / r^2) \tag{1}$$

In a superfluid the viscosity is zero, but there the imaginary quantity $\nu_{\mathrm{Q}} = i\hbar/m$ has the same dimension as a viscosity, and by naively replacing ν with ν_{Q}, one would have instead

$$v \propto \exp(-i \, \mathrm{const} \cdot |\nu_{\mathrm{Q}}| t / r^2) \tag{2}$$

Therefore, if (1) is the solution with the smallest decay rate for the value of $\mathrm{Re} \sim 10^6$, it is plausible that (2) is the corresponding minimum-frequency, and hence minimum-energy, solution for the quantum Reynolds number $\mathrm{Re}^{\mathrm{Q}} = irv/\nu_{\mathrm{Q}}$.

Applied to a vortex sponge, with each vortex having the potential flow solution of the form $v_\phi = cr_0/r$, we may assume that the velocity of the potential vortex has a cutoff at $r = r_0$ where $v_\phi = c$. In a fluid composed of particles (for example, superfluid helium) this cutoff length is in order of magnitude the mean free path λ and c is there the velocity of sound. Using the gas-kinetic relation $\nu \simeq c\lambda$, we find for the Reynolds number for the vortex core $\mathrm{Re} = c\lambda/\nu \simeq 1$. Outside the vortex core $\mathrm{curl}\,\mathbf{v} = 0$, and all the friction losses slowing down the vortex therefore come from the region within the vortex core. The effective viscosity averaged over the volume occupied by a line vortex within a vortex lattice of lattice constant r_1 is therefore $\bar{\nu} = c\lambda(\lambda/r_1)^2$, and the averaged Reynolds number is $\overline{\mathrm{Re}} = c\lambda/\bar{\nu} = (r_1/\lambda)^2$. Putting $\overline{\mathrm{Re}} \sim 10^6$, one would obtain $r_1 \sim 10^3\lambda$, which in order of magnitude agrees with the distance of separation between vortex filaments obtained by Schlayer (1928) for the Karman vortex street.

In analogy, one would expect for a vortex sponge of a superfluid that

$$\overline{\mathrm{Re}^{\mathrm{Q}}} = (r_1/r_0)^2 \tag{3}$$

If $\overline{\mathrm{Re}^{\mathrm{Q}}} \sim 10^6$, this would result in

$$r_0 \sim 10^{-33}\ \mathrm{cm} \tag{4}$$

which is of the same order of magnitude of the Planck length (which otherwise requires for its derivation the velocity of light and Planck and gravitational constants). With the quantum viscosity given by $|\nu_{\mathrm{Q}}| = \hbar/m_0$, one would have $m_0 = \hbar/r_0 c$, which is the Planck mass, if the velocity of sound is set equal to the velocity of light. Starting from the GUT scale at $r_1 \sim 10^{-30}$ cm, we therefore arrive at the Planck scale $r_0 \sim 10^{-3} r_1 \sim 10^{-33}$ cm solely by using fluid dynamic arguments. Beyond that it can be shown that the model even permits the derivation of the Maxwell and Einstein vacuum field equations. For material objects held together by these fields, Lorentz invariance then follows as a dynamic symmetry, very much as in the older, pre-Einstein theory of relativity of Lorentz and Poincaré.

3. ELECTROMAGNETIC AND GRAVITATIONAL WAVES

According to Helmholtz (1858), the most general displacement of a deformable body consists of (1) a translation, (2) a rotation, and (3) a strain. In a solid only the displacement by a strain can lead to a disturbance propagated as a wave, because only a strain generates there a stress acting against the deformation of the solid. In a vortex sponge, a rotational displacement can also lead to a wave. It was first recognized by Thomson (1887) that a vortex sponge permits the propagation of transverse waves due to rotational displacements, which for small amplitudes have the same property as the electromagnetic waves derived from Maxwell's equations. The other kind of wave possible in a vortex sponge is similar to a transverse wave in an elastic body. It is associated with an elliptic deformation of the vortex sponge, and can be identified with Einstein's gravitational waves. The strange difference in character between the electromagnetic and the gravitational fields has in the vortex sponge hypothesis therefore an almost trivial explanation, because, according to the theorem by Helmholtz, a disturbance of a body can, in general, always be decomposed into two irreducible parts of a symmetric and an antisymmetric tensor.

To analyze the transverse waves (Winterberg, 1990*a*), let $\mathbf{v} = \{v_x, v_y, v_z\}$ be the undisturbed velocity in the vortex sponge and $\mathbf{u} = \{u_x, u_y, u_z\}$ be a small superimposed velocity disturbance. Furthermore, let us take only those solutions for which $\operatorname{div} \mathbf{v} = \operatorname{div} \mathbf{u} = 0$. To reduce the problem to the solution of a differential equation, we must go to the continuum limit. This can be done by letting the vortex lattice constant go from r_1 to r_0, where r_0 is taken in the limit $r_0 \to 0$. The x component of the equation of motion for a disturbance **u** is

$$\frac{\partial v_x}{\partial t}+\frac{\partial u_x}{\partial t}=-(v_x+u_x)\frac{\partial(v_x+u_x)}{\partial x}-(v_y+u_y)\frac{\partial(v_x+u_x)}{\partial y}$$
$$-(v_z+u_z)\frac{\partial(v_x+u_x)}{\partial z}-\frac{1}{\rho}\frac{\partial p}{\partial x} \tag{5}$$

From the continuity equation div **v** = 0 we have

$$v_x\frac{\partial v_x}{\partial x}+v_x\frac{\partial v_y}{\partial y}+v_x\frac{\partial v_z}{\partial z}=0 \tag{6}$$

Subtracting (6) from (5) and taking the y-z average, we find

$$\frac{\partial u_x}{\partial t}=-\frac{\partial(\overline{v_y v_x})}{\partial y}-\frac{\partial(\overline{v_z v_x})}{\partial z} \tag{7a}$$

and similarly, by taking the x-z and x-y averages,

$$\frac{\partial u_y}{\partial t}=-\frac{\partial(\overline{v_x v_y})}{\partial x}-\frac{\partial(\overline{v_z v_y})}{\partial z} \tag{7b}$$

$$\frac{\partial u_z}{\partial t}=-\frac{\partial(\overline{v_x v_z})}{\partial x}-\frac{\partial(\overline{v_y v_z})}{\partial y} \tag{7c}$$

We note $\overline{v_x v_y} \neq \overline{v_y v_x}$, because for $\overline{v_x v_y}$ we took the x-z average, whereas for $\overline{v_y v_x}$ we took the y-z average. In general, $\overline{v_i v_k} \neq \overline{v_k v_i}$. With the condition div **u** = 0, we obtain from (7a)-(7c)

$$\overline{v_i v_k}=-\overline{v_k v_i} \tag{8}$$

Taking the x component of the equation of motion, multiplying it by v_y, and then taking the y-z average, and the y component multiplied by v_x and taking the x-z average, and finally subtracting the first from the second equation, we find

$$\frac{\partial}{\partial t}(\overline{v_x v_y})=-v^2\left(\frac{\partial u_y}{\partial x}-\frac{\partial u_x}{\partial y}\right) \tag{9}$$

where $v^2=\overline{v_x^2}=\overline{v_y^2}=\overline{v_z^2}$ is the average microvelocity of the vortex lattice. Putting $\phi_z=-\overline{v_x v_y}/2v^2$, we find that equation (9) is just the z component of

$$\frac{\partial \boldsymbol{\phi}}{\partial t}=\frac{1}{2}\operatorname{curl}\mathbf{u} \tag{10}$$

where $\phi_x = -\overline{v_y v_z}/2v^2$, $\phi_y = -\overline{v_z v_x}/2v^2$. Equations (7a)-(7c) then take the form

$$\frac{\partial \mathbf{u}}{\partial t} = -2v^2 \operatorname{curl} \boldsymbol{\phi} \tag{11}$$

Elimination of $\boldsymbol{\phi}$ from (10) and (11) results in a wave equation for $\mathbf{u}$,

$$-(1/v^2)\, \partial^2\mathbf{u}/\partial t^2 + \nabla^2\mathbf{u} = 0 \tag{12}$$

In the collapsed vortex lattice, making the transition $r_1 \to r_0$, one should have for the microvelocity $v^2 = c^2$. In this limit equation (12) describes a transverse wave propagating with the velocity of light c. In reality, though, $r_1 \gtrsim 10^3 r_0$, which means that the equation describing this wave would break down for wavelengths smaller than $\sim r_1$.

With $v = c$ and putting $\mathbf{u} = \mathbf{E}$ and $\boldsymbol{\phi} = -(1/2c)\mathbf{H}$, we find that (10) and (11) have the same form as the two Maxwell vacuum field equations

$$-\frac{1}{c}\frac{\partial \mathbf{H}}{\partial t} = \operatorname{curl} \mathbf{E} \tag{13}$$

$$\frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} = \operatorname{curl} \mathbf{H} \tag{14}$$

Adding (6) to (5) and taking the average over x, y, and z, we have

$$\frac{\partial u_x}{\partial t} = -\frac{\partial \overline{v_x^2}}{\partial x} - \frac{\partial \overline{v_x v_y}}{\partial y} - \frac{\partial \overline{v_x v_z}}{\partial z} \tag{15a}$$

and similarly

$$\frac{\partial u_y}{\partial t} = -\frac{\partial \overline{v_y^2}}{\partial y} - \frac{\partial \overline{v_y v_z}}{\partial z} - \frac{\partial \overline{v_y v_x}}{\partial x} \tag{15b}$$

$$\frac{\partial u_z}{\partial t} = -\frac{\partial \overline{v_z^2}}{\partial z} - \frac{\partial \overline{v_z v_x}}{\partial x} - \frac{\partial \overline{v_z v_y}}{\partial y} \tag{15c}$$

Combining (15a)-(15c) with the condition div $\mathbf{u}$ leads to

$$\frac{\partial^2}{\partial x_i\, \partial x_k}(\overline{v_i v_k}) = 0 \tag{16}$$

and for (15a)-(15c) we can write

$$\frac{\partial u_k}{\partial t} = -\frac{\partial}{\partial x_i}(\overline{v_i v_k}) \tag{17}$$

Multiplying the v_i component of the equation of motion with v_k, its v_k component with v_i, adding both, and taking the average, we find

$$\frac{\partial}{\partial t}(\overline{v_i v_k}) = -v^2\left(\frac{\partial u_i}{\partial x_k}+\frac{\partial u_k}{\partial x_i}\right) \tag{18}$$

From (17) we have

$$\frac{\partial^2 u_k}{\partial t^2} = -\frac{\partial}{\partial t\,\partial x_i}(\overline{v_i v_k}) \tag{19}$$

and from (18)

$$\frac{\partial}{\partial x_i\,\partial t}(\overline{v_i v_k}) = -v^2\left(\frac{\partial}{\partial x_k}\frac{\partial u_i}{\partial x_i}+\frac{\partial^2 u_k}{\partial x_i^2}\right) = -v^2\frac{\partial^2 u_k}{\partial x_i^2} \tag{20}$$

the latter because of $\operatorname{div}\mathbf{u}=0$. Eliminating $\overline{v_i v_k}$ from (19) and (20) and putting as before $v^2=c^2$ finally results in

$$\frac{\partial^2 u_k}{\partial t^2} = c^2\frac{\partial^2 u_k}{\partial x_i^2} \tag{21}$$

or

$$\nabla^2\mathbf{u}-\frac{1}{c^2}\frac{\partial^2\mathbf{u}}{\partial t^2}=0 \tag{22}$$

We claim that (22) can be interpreted as a linearized gravitational wave equation derived from Einstein's gravitational field theory. To demonstrate this equivalence, we consider a gravitational wave propagating in the x direction. It is described by the following line element (Landau and Lifshitz, 1975):

$$ds^2 = ds_0^2 + h_{22}\,dx_2^2 + 2h_{23}\,dx_2\,dx_3 + h_{33}\,dx_3^2 \tag{23}$$

where

$$\begin{aligned} h_{22} &= -h_{33} = f(t-x/c) \\ h_{23} &= g(t-x/c) \end{aligned} \tag{24}$$

with f and g two arbitrary functions, and ds_0^2 the line element in the absence of a gravitational wave. A deformation of an elastic body can likewise be described by a distorted line element as follows (Landau and Lifshitz, 1970):

$$ds^2 = ds_0^2 + 2\varepsilon_{ik}\,dx_i\,dx_k; \qquad i,\ k=1,2,3 \tag{25}$$

where

$$\varepsilon_{ik} = \frac{1}{2}\left(\frac{\partial \varepsilon_i}{\partial x_k}+\frac{\partial \varepsilon_k}{\partial x_i}\right) \tag{26}$$

In (26), $\boldsymbol{\varepsilon} = (\varepsilon_x, \varepsilon_y, \varepsilon_x)$ is the displacement vector, which is related to the velocity disturbance vector $\mathbf{u}$ by

$$\mathbf{u} = \frac{\partial \boldsymbol{\varepsilon}}{\partial t} \tag{27}$$

In an elastic medium, transverse waves obey the wave equation

$$\nabla^2 \boldsymbol{\varepsilon} - \frac{1}{c^2} \frac{\partial^2 \boldsymbol{\varepsilon}}{\partial t^2} = 0 \tag{28}$$

which because of (27) is the same as (22). From the condition $\operatorname{div} \mathbf{u} = 0$ and (27) it follows $\operatorname{div} \boldsymbol{\varepsilon} = 0$.

For a transverse wave propagating into the x direction, $\varepsilon_x = \varepsilon_1 = 0$ and the condition $\operatorname{div} \boldsymbol{\varepsilon} = 0$ leads to

$$\frac{\partial \varepsilon_2}{\partial x_2} + \frac{\partial \varepsilon_3}{\partial x_3} = \varepsilon_{22} + \varepsilon_{33} = 0 \tag{29}$$

Hence

$$\varepsilon_{33} = -\varepsilon_{22} \tag{30}$$

For the identification with a gravitational wave one has to put

$$2\varepsilon_{ik} = h_{ik} \tag{31}$$

Figure 1 illustrates how an electromagnetic and a gravitational wave distort a vortex lattice, composed of vortex rings.

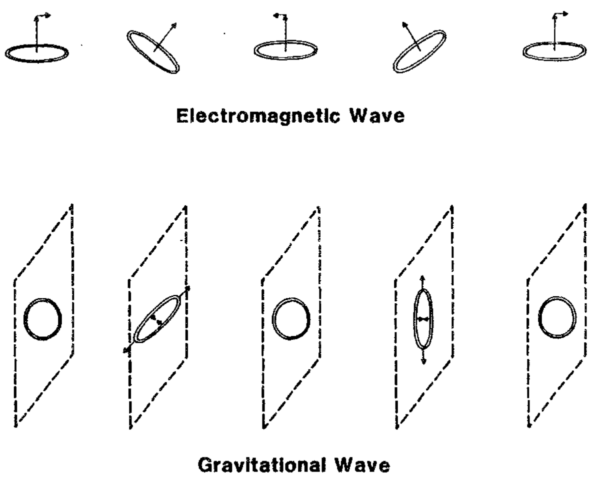

Fig. 1. Deformation of the vortex lattice for an electromagnetic wave and a gravitational wave.

4. LORENTZ INVARIANCE

If solid bodies are held together by electromagnetic forces, or forces acting like them, Lorentz invariance can be explained as a dynamic symmetry, because if all interactions holding the body together behave like the electromagnetic interactions, all clocks should behave like light clocks, and the combined effect of the Lorentz contraction and anisotropic light propagation in a moving frame makes a light clock move slower by the same factor $(1-u^2/c^2)^{1/2}$ as in special relativity (Prokhovnik, 1967). The Lorentz contraction alone is therefore sufficient to derive the Lorentz transformations as a dynamic symmetry for objects in a state of internal equilibrium. How Lorentz transformations can be interpreted as Galilei transformations with physical length and time contractions has been shown by Wilhelm (1988).

5. NONLINEARITY OF GRAVITATIONAL FIELD

The nonlinearity of the gravitational field, which at first sight does not seem to be apparent from the linear gravitational wave solution (24), follows from an argument by Gupta (1954). By a space-time coordinate transformation, the wave equation

$$\left(-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}+\frac{\partial^2}{\partial x^2}\right)h_{ik}=0, \qquad i, k=1,2,3,4 \tag{32}$$

can be brought into the form (Landau and Lifshitz, 1975)

$$\Box\psi_i^k=0, \qquad i, k=1,2,3,4 \tag{33}$$

with the subsidiary (gauge) condition

$$\frac{\partial\psi_i^k}{\partial x^k}=0 \tag{34}$$

where the tensor ψ_i^k represents the gravitational field. According to the minimum coupling principle, the gravitational field equation in the presence of matter has the form

$$\Box\psi_i^k=\varkappa\Theta_i^k, \qquad \varkappa=\text{const} \tag{35}$$

where Θ_i^k is a four-dimensional, relativistically invariant symmetric tensor which because of (34) obeys the conservation equation

$$\frac{\partial\Theta_i^k}{\partial x^k}=0 \tag{36}$$

The only physically possible invariant tensor obeying Lorentz invariance as a dynamic symmetry and satisfying the conservation law (36) is the total energy-momentum tensor of matter and gravitational field. As was shown by Gupta, by splitting Θ_i^k into a matter part T_i^k and a gravitational field part t_i^k,

$$\Theta_i^k = T_i^k + t_i^k \tag{37}$$

we can bring the field equation (35) into Einstein's form

$$R_{ik} - \tfrac{1}{2} g_{ik} R = \varkappa T_{ik} \tag{38}$$

where it is expressed as a field equation in a non-Euclidean space-time manifold. According to our model, where space is Euclidean and time absolute, with special relativity caused by true physical deformations, the reason why the field equation can be formulated by a non-Euclidean manifold has nothing to do with a supposedly curved space-time. It rather results from the principle of equivalence, whereby all bodies, given the same initial conditions, follow the same trajectory. As in the dynamic interpretation of special relativity, where the Minkowskian space-time manifold is seen as an illusion caused by a true deformation of bodies, a Riemannian manifold must here be seen as an illusion as well, caused by true physical deformations in conjunction with the illusion generated by the nonlinearity of the field equations.

6. THE ORIGIN OF CHARGE

The phenomenon of charge is explained in the model as follows: At the scale r_0, masses m_0 are bound in the core of vortex filaments of radius r_0. According to the uncertainty principle, they have the zero-point energy

$$m_0 c^2 \simeq \frac{\hbar c}{2 r_0} \tag{39}$$

In the volume $(4\pi/3) r_0^3$ the kinetic energy density due to this zero-point fluctuation is

$$\varepsilon_k \simeq \frac{\hbar c}{8 r_0^4} \tag{40}$$

The zero-point energy of the mass m_0 therefore leads to an attractive inverse-square-law field produced by virtual phonons, having their source in the oscillatory zero-point motion of the mass m_0. If this field has the strength F, the field energy density at the distance r_0 is

$$\varepsilon_f = \frac{F^2}{4\pi} = \frac{g^2}{4\pi r_0^4} \tag{41}$$

where g is the coupling constant of the phonons to the mass m_0. Equating

(40) with (41), one finds

$$g^2 \simeq \hbar c \tag{42}$$

In a similar manner, electric charge can be explained as resulting from zero-point directional fluctuations in the vortex sponge, and gravitational charge as resulting from the zero-point deformation fluctuations.

Solitons describing elementary particles and made up from excitations of the vortex sponge would interact with a coupling constant of the order $g \simeq (\hbar c)^{1/2}$, and Lorentz invariance as a dynamic symmetry would be valid for them. Because $Gm_0^2 = \hbar c$, (39) is of equal magnitude to the gravitational interaction energy of two masses m_0 separated by the distance r_0.

7. DIRAC SPINORS

To explain Dirac spinors with this model is more difficult, but even this can be done by an extension of the model (Winterberg, 1988, 1990*b*). It goes as follows: With the zero-point energy cutoff at the Planck energy $m_0c^2 \simeq 10^{19}$ GeV, the vacuum mass density would be

$$\rho_0 \sim c^5/\hbar G^2 \sim 10^{95}\ \mathrm{g/cm^3} \tag{43}$$

large enough to put the mass of the entire universe in a cube with the side length of 1 fermi. To overcome this problem, it was suggested many years ago by Sakharov (1968) that there are compensating negative-energy "ghost particles." Sakharov's idea lends itself to the pole–dipole particle model of Hönl and Papapetrou (1939). It can reproduce Schrödinger's (1930, 1931) "Zitterbewegung" of the Dirac spinor, and hence the spin. As shown in Figure 2, the pole–dipole particle is made up of a positive mass m^+ coupled by an attractive force to a negative "ghost" mass m^-. In the presence of an attractive force the two masses execute a circular motion around their center of mass. In case one of the masses is negative, but with both together having a positive mass pole $m_p = m^+ - |m^-|$, the circular motion persists, except that the center of mass is no longer between the masses, even though it is still located on the line connecting m^+ and m^-. It is the rotational motion which causes the spin, and it has the same property as the Zitterbewegung.

If $|m^+| > |m^-|$, the distance of m^- from the center of mass is larger than for m^+. We assume that m^+ is at a distance r_c, with m^- at a distance $r_c + r$. Furthermore, if $m_p \ll m^+ \simeq |m^-|$, one has $r \ll r_c$. Defining $\gamma_+ = (1 - v^2/c^2)^{-1/2}$, with $v = r_c\omega$, where ω is the angular velocity around the center of mass, and $\gamma_- = (1 - v_-^2/c^2)^{-1/2}$, with $v_- = (r_c + r)\omega$, we find that momentum conservation leads to

$$m^+\gamma_+ r_c = |m^-|\gamma_-(r_c + r) \tag{44}$$

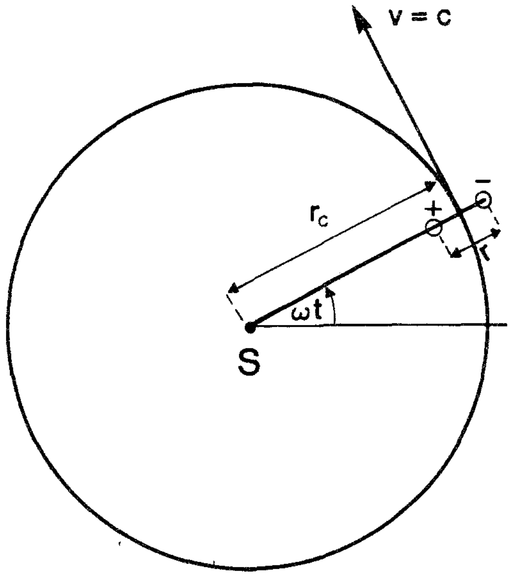

Fig. 2. Pole–dipole particle.

For $r \ll r_c$ we expand

$$\gamma_- = \gamma\left(1 + \frac{r_c r \omega^2 \gamma^2}{c^2} + \cdots\right) \tag{45}$$

putting henceforth $\gamma_+ \equiv \gamma$.

For the mass dipole moment we have

$$p = m^+ r \simeq |m^-| r = \frac{m^+ \gamma - |m^-| \gamma_-}{\gamma_-} r_c \tag{46}$$

With the help of (45) and for $\gamma \gg 1$ we find

$$r_c \simeq p\gamma^2 / m_p \tag{47}$$

for the energy we find

$$E/c^2 = m = m^+ \gamma - |m^-| \gamma_- \simeq p\gamma / r_c \tag{48}$$

and finally, for the angular momentum (putting $\omega r_c \simeq c$)

$$J = [m^+ \gamma r_c^2 - |m^-| \gamma_- (r_c + r)^2] \omega \simeq -p\gamma c \simeq -mcr_c \tag{49}$$

Let us now assume that the two masses m^+ and m^- are initially exactly equal, but of opposite sign. According to what we said above, the interaction energy of two Planck masses m_0 is equal in magnitude to their gravitational interaction at the distance of separation r_0, which is equal to the Planck length. Since the virtual phonon force field causing this interaction follows an inverse square law, and since the gravitational interaction of two masses of opposite sign is positive, one may see what would happen if the mass pole of the pole-dipole particle comes from this interaction energy. One finds

$$m_p c^2 = G|m^{\pm}|^2/r \tag{50}$$

but because of (47) and (48) one has[2]

$$m = m_p/\gamma \tag{51}$$

and hence

$$mc^2 = G|m^{\pm}|^2/\gamma r \tag{52}$$

According to the correspondence principle, for angular momentum quantization one should put[3] $J = -\hbar$, resulting, from (49), in $r_c = \hbar/mc$. Putting $p \simeq |m^{\pm}|r$, one obtains from (48)

$$\gamma|m^{\pm}|rc = \hbar \tag{53}$$

Finally, eliminating γr from (52) and (53) and solving for $|m^{\pm}|$, one finds

$$|m^{\pm}| = (\hbar mc/G)^{1/3} \tag{54}$$

Inserting into (54) for m the value of the electron mass, one has

$$|m^{\pm}| \simeq 8 \times 10^{-13}\ \mathrm{g} \tag{55}$$

hence $|m^{\pm}|c^2 \sim 10^{12}$ GeV. This is a very large energy, but there is widespread belief in the existence of such an intermediate energy located between the electroweak and GUT energies.

8. ORIGIN OF INERTIA AND MACH'S PRINCIPLE

Finally, our model is even able to make plausible the origin of inertia and through it the equivalence principle. In general relativity inertia is explained as a restraint force resulting from the restraint imposed by a noninertial reference system and expressed through Christoffel symbols. In the vortex sponge something like inertia results from the constraint $\operatorname{div} \mathbf{v} = 0$ of an incompressible fluid. Applying Newton's law of motion, with the force density acting on a fluid element equal to $-\nabla p$, we obtain Euler's equation

$$\frac{d\mathbf{v}}{dt} = -\frac{1}{\rho}\nabla p \tag{56}$$

[2]The mass m of the circulating pole-dipole particle is m_p/γ rather than $m_p\gamma$ because the orbital radius of the negative mass is larger by the distance r than its positive counterpart.

[3]This value is suggested by the correspondence principle, whereas relatively leads to $J = -(1/2)\hbar$.

which for an incompressible fluid has to be supplemented by the incompressibility condition $\operatorname{div} \mathbf{v} = 0$. Taking the divergence of Euler's equation, we obtain

$$\nabla^2\left(\frac{p}{\rho}+\frac{v^2}{2}\right) = \operatorname{div}(\mathbf{v} \times \operatorname{curl} \mathbf{v}) \tag{57}$$

and hence

$$\frac{p}{\rho} = -\frac{v^2}{2} - \int \frac{\operatorname{div}(\mathbf{v} \times \operatorname{curl} \mathbf{v})}{4\pi|\mathbf{r}-\mathbf{r}'|} \, d\mathbf{r}' \tag{58}$$

With the constraint $\operatorname{div} v = 0$ one can therefore eliminate the pressure from Euler's equation, which thereby becomes the following integrodifferential equation:

$$\frac{d\mathbf{v}}{dt} = \nabla\left(\frac{v^2}{2}\right) + \nabla \int \frac{\operatorname{div}(\mathbf{v} \times \operatorname{curl} \mathbf{v})}{4\pi|\mathbf{r}-\mathbf{r}'|} \, d\mathbf{r}' \tag{59}$$

Unlike Euler's equation, it is independent of the mass density, suggesting a purely kinematic interpretation, very much like the equation of motion for a test body in general relativity, where the trajectory the test body follows does not depend on its mass. Because the integral in (59) expresses the instantaneous interaction with the vortex sponge filling all of space, it is reminiscent of Mach's principle for the origin of inertia.

The surprising multifaced success of this simple vacuum model gives us reason to believe that "In the beginning was (quantum) chaos."

REFERENCES

Gupta, S. N. (1954). *Physical Review*, **96**, 1683.

Helmholtz v., H. (1858). *Crelles Journal*, **55**, 25.

Hönl, H., and Papapetrou, A. (1939). *Zeitschrift für Physik*, **112**, 512.

Landau, L. D., and Lifshitz, E. M. (1970). *Theory of Elasticity*, Pergamon Press, New York, p. 2.

Landau, L. D., and Lifshitz, E. M. (1975). *The Classical Theory of Fields*, Pergamon Press, New York.

Prokhovnik, S. J. (1967). *The Logic of Special Relativity*, Cambridge University Press, Cambridge.

Sakharov, A. D. (1968). *Soviet Physics Doklady*, **12**, 1040.

Schlayer, K. (1928). *Zeitschrift für Angewandte Mathematik und Mechanik*, **8**, 352.

Schrödinger, E. (1930). *Berliner Berichte*, **1930**, 415.

Schrödinger, E. (1931). *Berliner Berichte*, **1931**, 418.

Thomson, W. (Lord Kelvin) (1887). *Philosophical Magazine*, **24**, 342.

Wilhelm, H. E. (1988). *Zeitschrift für Naturforschung*, **43a**, 859.

Winterberg, F. (1988). *Zeitschrift für Naturforschung*, **43a**, 1131.

Winterberg, F. (1990*a*). *Zeitschrift für Naturforschung*, **45a**, 1102.

Winterberg, F. (1990*b*). *Zeitschrift für Naturforschung*, **46a**, 677.